



ÖVNINGAR PÅ TRIGONOMETRISKA FUNKTIONER

EUROPASKOLAN
STRÄNGNÄS

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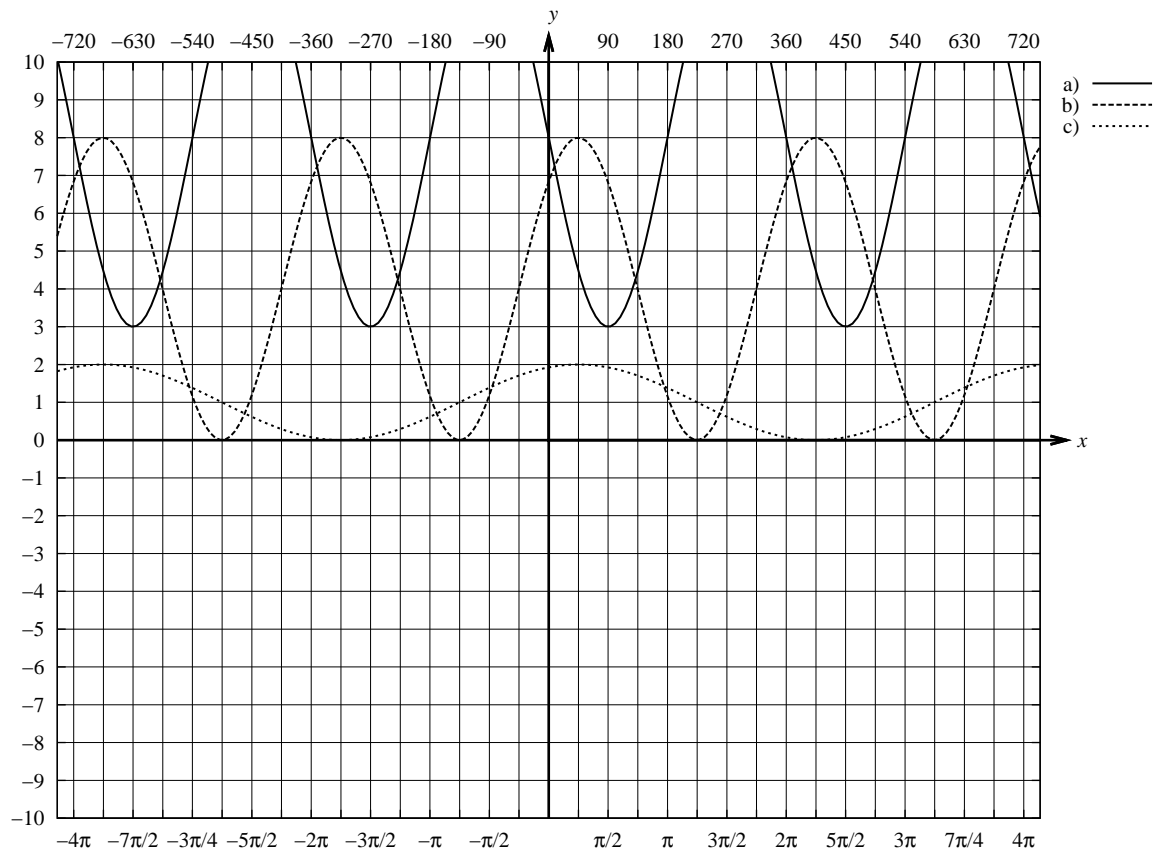
Här finns ett antal funktioner som beskriver trigonometriska funktioner och ett antal grafer. Öva på att rita grafen till funktionen $y(x)$ samt på att bestämma uttrycket för en graf.

Ledning: En trigonometrisk funktion kan skrivas på formen $y = A \sin(k(x - x_0)) + d$. Bestäm A och d direkt ur grafen. Leta upp en punkt där sinuskurvan "börjar" för att hitta x_0 . Identifiera hur lång en "våglängd" L är. Vågtalet k ges av $k \cdot L = 2\pi$ eller $k \cdot L = 360^\circ$.

- | | | |
|---|--|--|
| 1. a) $\begin{cases} 5 \sin(x + 180) + 8 \\ 5 \sin(x + \pi) + 8 \end{cases}$ | b) $\begin{cases} 4 \sin(x + 45) + 4 \\ 4 \sin(x + \frac{1}{4}\pi) + 4 \end{cases}$ | c) $\begin{cases} \sin(\frac{1}{2}(x + 135)) + 1 \\ \sin(\frac{1}{2}(x + \frac{3}{4}\pi)) + 1 \end{cases}$ |
| 2. a) $\begin{cases} 5 \sin(\frac{1}{2}(x - 135)) + 8 \\ 5 \sin(\frac{1}{2}(x - \frac{3}{4}\pi)) + 8 \end{cases}$ | b) $\begin{cases} \sin(\frac{1}{2}(x - 90)) - 8 \\ \sin(\frac{1}{2}(x - \frac{1}{2}\pi)) - 8 \end{cases}$ | c) $\begin{cases} 3 \sin(\frac{1}{2}(x - 270)) + 7 \\ 3 \sin(\frac{1}{2}(x - \frac{3}{2}\pi)) + 7 \end{cases}$ |
| 3. a) $\begin{cases} \sin(2(x + 90)) - 3 \\ \sin(2(x + \frac{1}{2}\pi)) - 3 \end{cases}$ | b) $\begin{cases} 5 \sin(3(x + 225)) - 5 \\ 5 \sin(3(x + \frac{5}{4}\pi)) - 5 \end{cases}$ | c) $\begin{cases} 4 \sin(2(x + 270)) + 4 \\ 4 \sin(2(x + \frac{3}{2}\pi)) + 4 \end{cases}$ |
| 4. a) $\begin{cases} 4 \sin(\frac{1}{3}x) - 6 \\ 4 \sin(\frac{1}{3}x) - 6 \end{cases}$ | b) $\begin{cases} 5 \sin(2(x - 90)) \\ 5 \sin(2(x - \frac{1}{2}\pi)) \end{cases}$ | c) $\begin{cases} 5 \sin(2(x + 225)) + 7 \\ 5 \sin(2(x + \frac{5}{4}\pi)) + 7 \end{cases}$ |
| 5. a) $\begin{cases} 3 \sin(x + 135) + 4 \\ 3 \sin(x + \frac{3}{4}\pi) + 4 \end{cases}$ | b) $\begin{cases} 5 \sin(x - 90) - 6 \\ 5 \sin(x - \frac{1}{2}\pi) - 6 \end{cases}$ | c) $\begin{cases} 4 \sin(2(x + 90)) + 8 \\ 4 \sin(2(x + \frac{1}{2}\pi)) + 8 \end{cases}$ |
| 6. a) $\begin{cases} 2 \sin(x + 270) + 1 \\ 2 \sin(x + \frac{3}{2}\pi) + 1 \end{cases}$ | b) $\begin{cases} \sin(3(x + 225)) + 2 \\ \sin(3(x + \frac{5}{4}\pi)) + 2 \end{cases}$ | c) $\begin{cases} 4 \sin(x - 135) - 5 \\ 4 \sin(x - \frac{3}{4}\pi) - 5 \end{cases}$ |
| 7. a) $\begin{cases} 5 \sin(x + 225) - 5 \\ 5 \sin(x + \frac{5}{4}\pi) - 5 \end{cases}$ | b) $\begin{cases} 2 \sin(\frac{1}{3}(x + 135)) - 4 \\ 2 \sin(\frac{1}{3}(x + \frac{3}{4}\pi)) - 4 \end{cases}$ | c) $\begin{cases} 2 \sin(x - 225) - 8 \\ 2 \sin(x - \frac{5}{4}\pi) - 8 \end{cases}$ |
| 8. a) $\begin{cases} \sin(\frac{1}{3}(x - 135)) + 8 \\ \sin(\frac{1}{3}(x - \frac{3}{4}\pi)) + 8 \end{cases}$ | b) $\begin{cases} \sin(x - 45) + 5 \\ \sin(x - \frac{1}{4}\pi) + 5 \end{cases}$ | c) $\begin{cases} 5 \sin(x - 90) - 8 \\ 5 \sin(x - \frac{1}{2}\pi) - 8 \end{cases}$ |
| 9. a) $\begin{cases} 4 \sin(x - 270) + 2 \\ 4 \sin(x - \frac{3}{2}\pi) + 2 \end{cases}$ | b) $\begin{cases} 4 \sin(3(x + 135)) - 1 \\ 4 \sin(3(x + \frac{3}{4}\pi)) - 1 \end{cases}$ | c) $\begin{cases} 4 \sin(\frac{1}{2}(x + 90)) + 6 \\ 4 \sin(\frac{1}{2}(x + \frac{1}{2}\pi)) + 6 \end{cases}$ |
| 10. a) $\begin{cases} \sin(2(x - 225)) - 3 \\ \sin(2(x - \frac{5}{4}\pi)) - 3 \end{cases}$ | b) $\begin{cases} 2 \sin(\frac{1}{2}(x + 270)) + 1 \\ 2 \sin(\frac{1}{2}(x + \frac{3}{2}\pi)) + 1 \end{cases}$ | c) $\begin{cases} 4 \sin(\frac{1}{2}(x + 45)) - 4 \\ 4 \sin(\frac{1}{2}(x + \frac{1}{4}\pi)) - 4 \end{cases}$ |
| 11. a) $\begin{cases} \sin(x - 225) - 6 \\ \sin(x - \frac{5}{4}\pi) - 6 \end{cases}$ | b) $\begin{cases} 3 \sin(3(x + 270)) + 2 \\ 3 \sin(3(x + \frac{3}{2}\pi)) + 2 \end{cases}$ | c) $\begin{cases} 3 \sin(x + 90) - 5 \\ 3 \sin(x + \frac{1}{2}\pi) - 5 \end{cases}$ |
| 12. a) $\begin{cases} 4 \sin(x) + 5 \\ 4 \sin(x) + 5 \end{cases}$ | b) $\begin{cases} \sin(\frac{1}{2}(x + 135)) - 8 \\ \sin(\frac{1}{2}(x + \frac{3}{4}\pi)) - 8 \end{cases}$ | c) $\begin{cases} \sin(x - 180) - 4 \\ \sin(x - \pi) - 4 \end{cases}$ |
| 13. a) $\begin{cases} 3 \sin(x - 180) \\ 3 \sin(x - \pi) \end{cases}$ | b) $\begin{cases} 5 \sin(\frac{1}{3}x) + 7 \\ 5 \sin(\frac{1}{3}x) + 7 \end{cases}$ | c) $\begin{cases} \sin(2(x - 180)) + 1 \\ \sin(2(x - \pi)) + 1 \end{cases}$ |

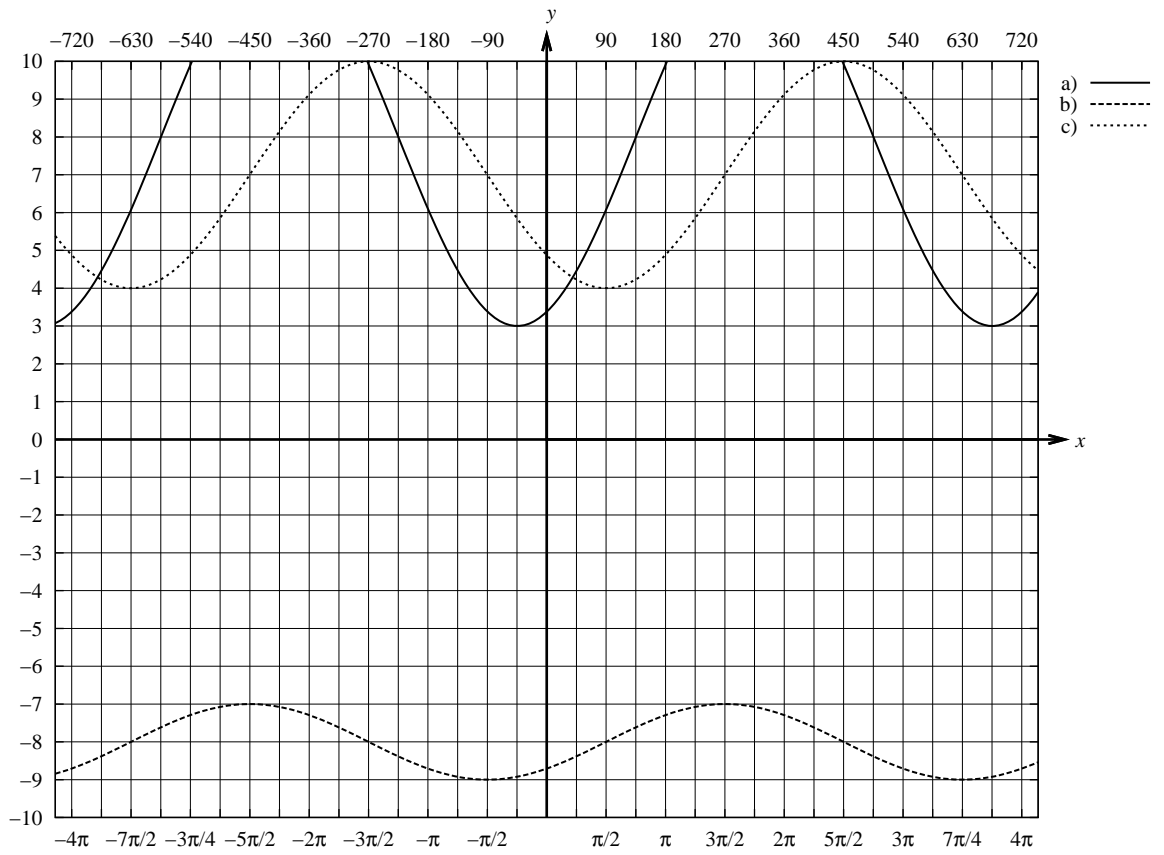
14. a) $\begin{cases} 5 \sin(2(x + 225)) - 7 \\ 5 \sin(2(x + \frac{5}{4}\pi)) - 7 \end{cases}$ b) $\begin{cases} 2 \sin(\frac{1}{3}(x - 270)) + 2 \\ 2 \sin(\frac{1}{3}(x - \frac{3}{2}\pi)) + 2 \end{cases}$ c) $\begin{cases} \sin(x + 90) + 3 \\ \sin(x + \frac{1}{2}\pi) + 3 \end{cases}$
15. a) $\begin{cases} 3 \sin(\frac{1}{2}(x - 180)) - 8 \\ 3 \sin(\frac{1}{2}(x - \pi)) - 8 \end{cases}$ b) $\begin{cases} \sin(x + 270) + 2 \\ \sin(x + \frac{3}{2}\pi) + 2 \end{cases}$ c) $\begin{cases} 4 \sin(x - 45) - 7 \\ 4 \sin(x - \frac{1}{4}\pi) - 7 \end{cases}$
16. a) $\begin{cases} 4 \sin(\frac{1}{2}(x - 225)) - 1 \\ 4 \sin(\frac{1}{2}(x - \frac{5}{4}\pi)) - 1 \end{cases}$ b) $\begin{cases} 5 \sin(2(x + 135)) - 7 \\ 5 \sin(2(x + \frac{3}{4}\pi)) - 7 \end{cases}$ c) $\begin{cases} 4 \sin(3x) - 8 \\ 4 \sin(3x) - 8 \end{cases}$
17. a) $\begin{cases} 3 \sin(\frac{1}{3}x) - 2 \\ 3 \sin(\frac{1}{3}x) - 2 \end{cases}$ b) $\begin{cases} 5 \sin(3(x + 180)) \\ 5 \sin(3(x + \pi)) \end{cases}$ c) $\begin{cases} 2 \sin(\frac{1}{2}(x - 45)) - 3 \\ 2 \sin(\frac{1}{2}(x - \frac{1}{4}\pi)) - 3 \end{cases}$
18. a) $\begin{cases} 5 \sin(2(x - 225)) + 3 \\ 5 \sin(2(x - \frac{5}{4}\pi)) + 3 \end{cases}$ b) $\begin{cases} 5 \sin(x - 270) - 2 \\ 5 \sin(x - \frac{3}{2}\pi) - 2 \end{cases}$ c) $\begin{cases} 5 \sin(3(x - 225)) - 7 \\ 5 \sin(3(x - \frac{5}{4}\pi)) - 7 \end{cases}$
19. a) $\begin{cases} 2 \sin(3(x - 45)) - 3 \\ 2 \sin(3(x - \frac{1}{4}\pi)) - 3 \end{cases}$ b) $\begin{cases} \sin(2(x + 135)) \\ \sin(2(x + \frac{3}{4}\pi)) \end{cases}$ c) $\begin{cases} 3 \sin(\frac{1}{3}(x + 225)) - 7 \\ 3 \sin(\frac{1}{3}(x + \frac{5}{4}\pi)) - 7 \end{cases}$
20. a) $\begin{cases} 4 \sin(2(x + 180)) - 1 \\ 4 \sin(2(x + \pi)) - 1 \end{cases}$ b) $\begin{cases} \sin(\frac{1}{2}(x + 45)) + 1 \\ \sin(\frac{1}{2}(x + \frac{1}{4}\pi)) + 1 \end{cases}$ c) $\begin{cases} \sin(\frac{1}{2}(x - 45)) + 6 \\ \sin(\frac{1}{2}(x - \frac{1}{4}\pi)) + 6 \end{cases}$
21. a) $\begin{cases} 5 \sin(\frac{1}{3}(x + 270)) + 4 \\ 5 \sin(\frac{1}{3}(x + \frac{3}{2}\pi)) + 4 \end{cases}$ b) $\begin{cases} 4 \sin(2(x + 225)) - 3 \\ 4 \sin(2(x + \frac{5}{4}\pi)) - 3 \end{cases}$ c) $\begin{cases} 4 \sin(2(x + 270)) - 5 \\ 4 \sin(2(x + \frac{3}{2}\pi)) - 5 \end{cases}$

Grafer:

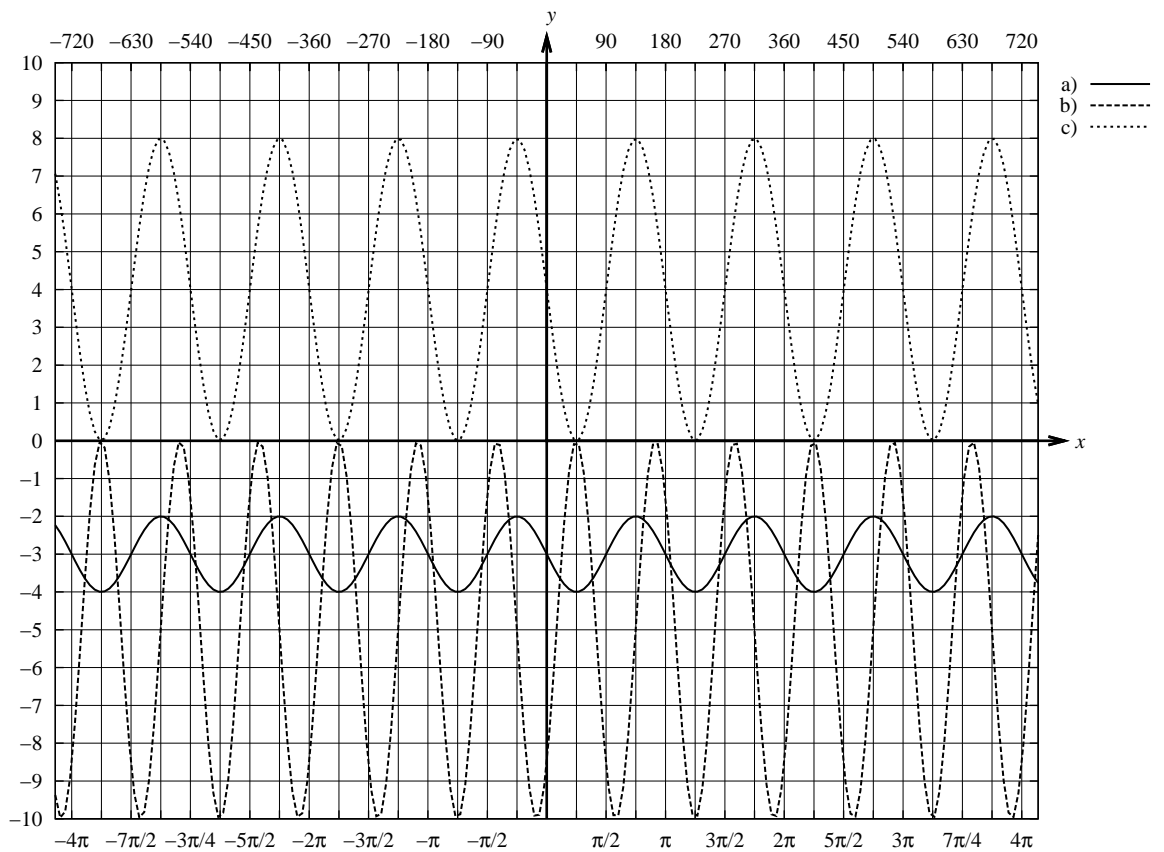


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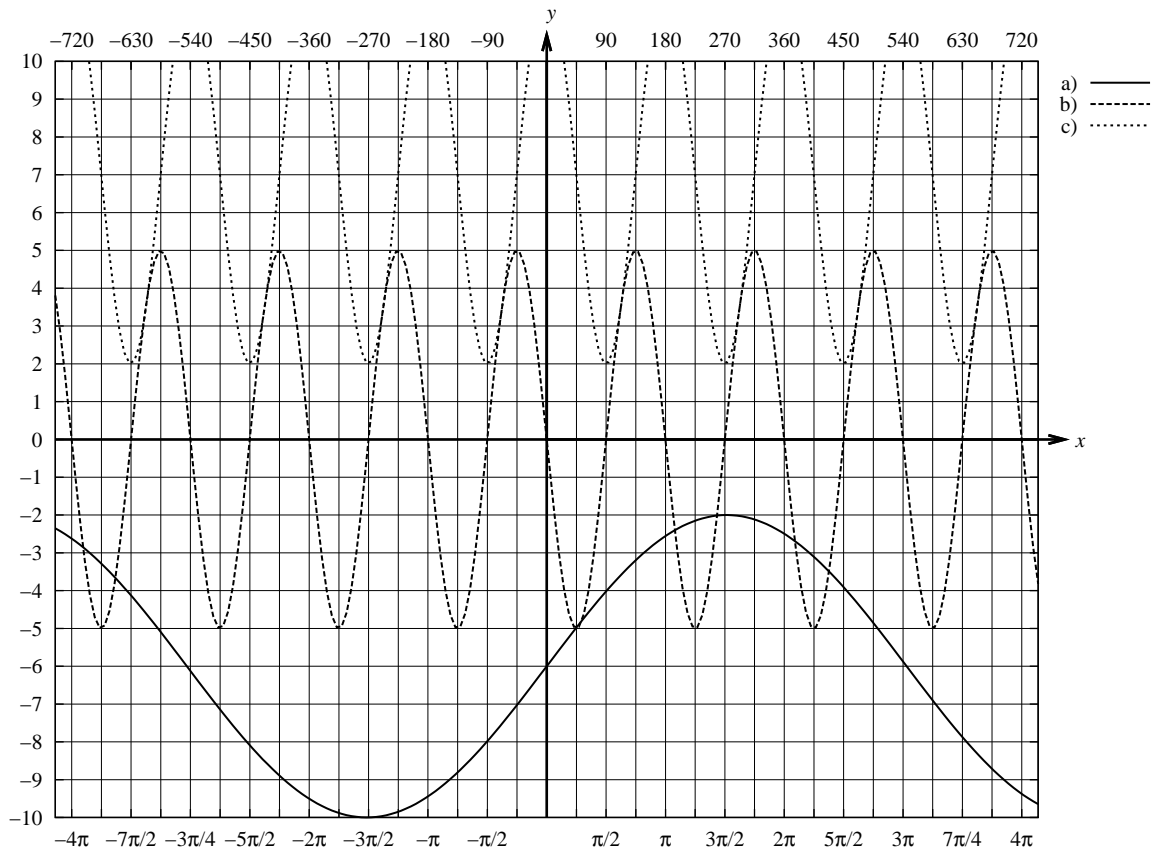
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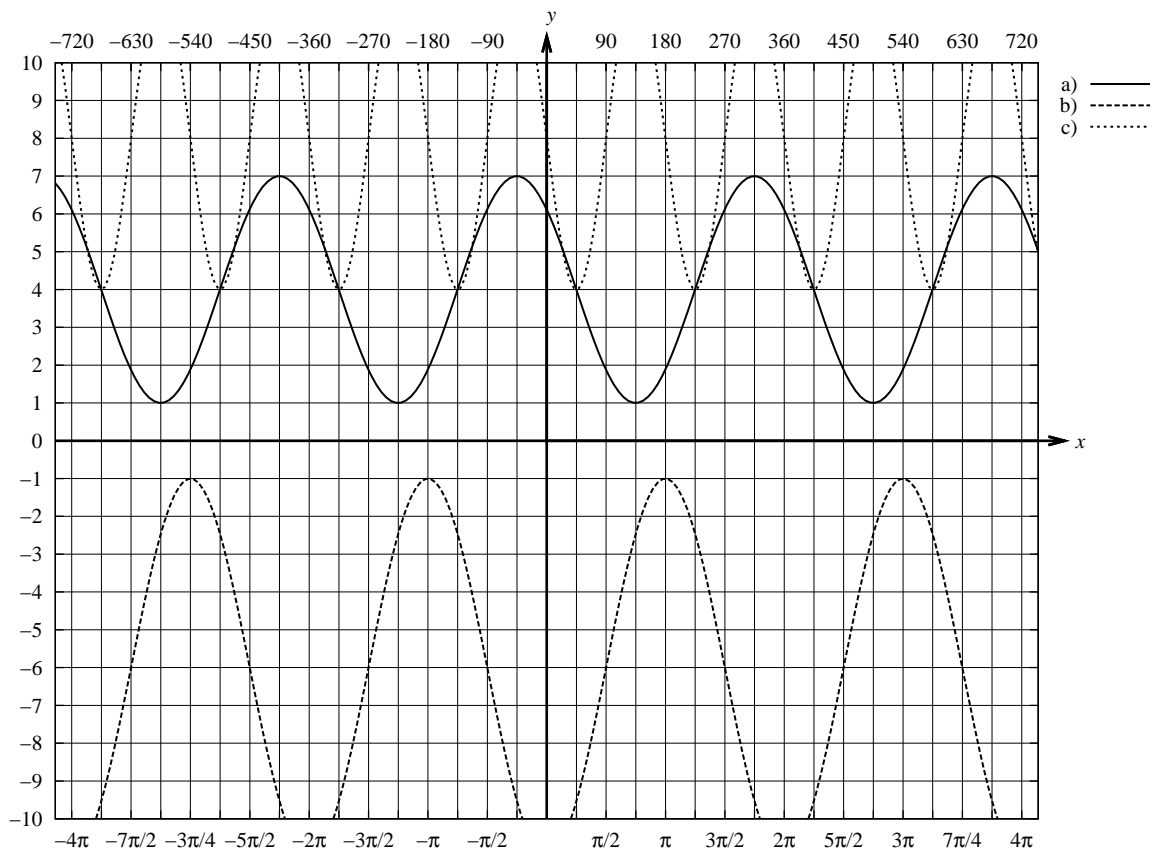
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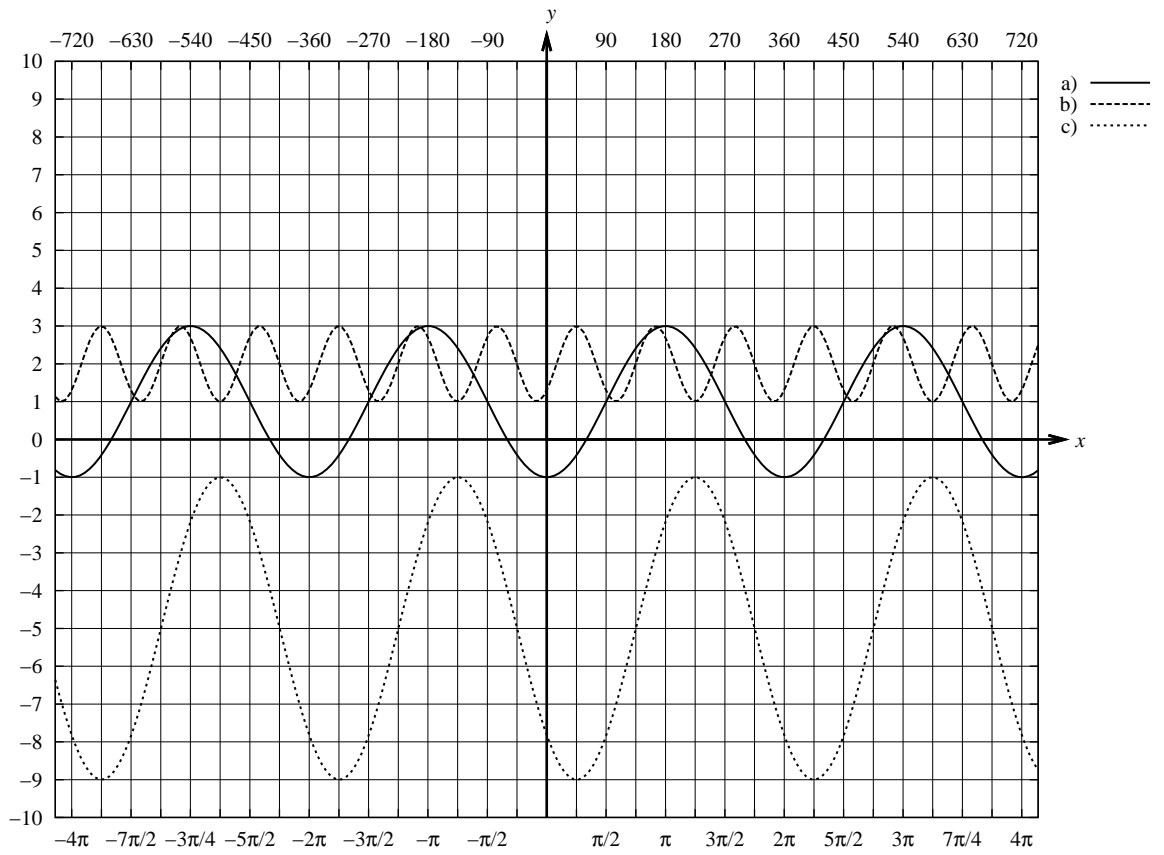
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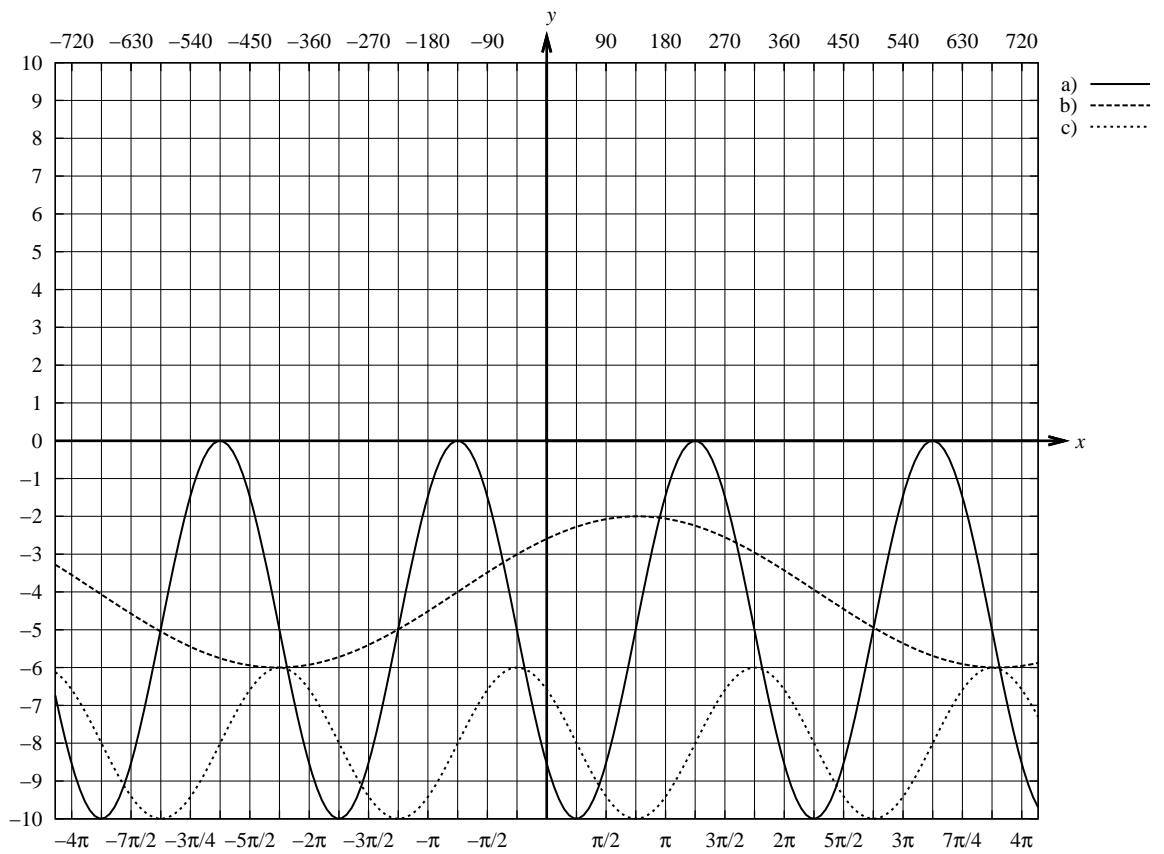
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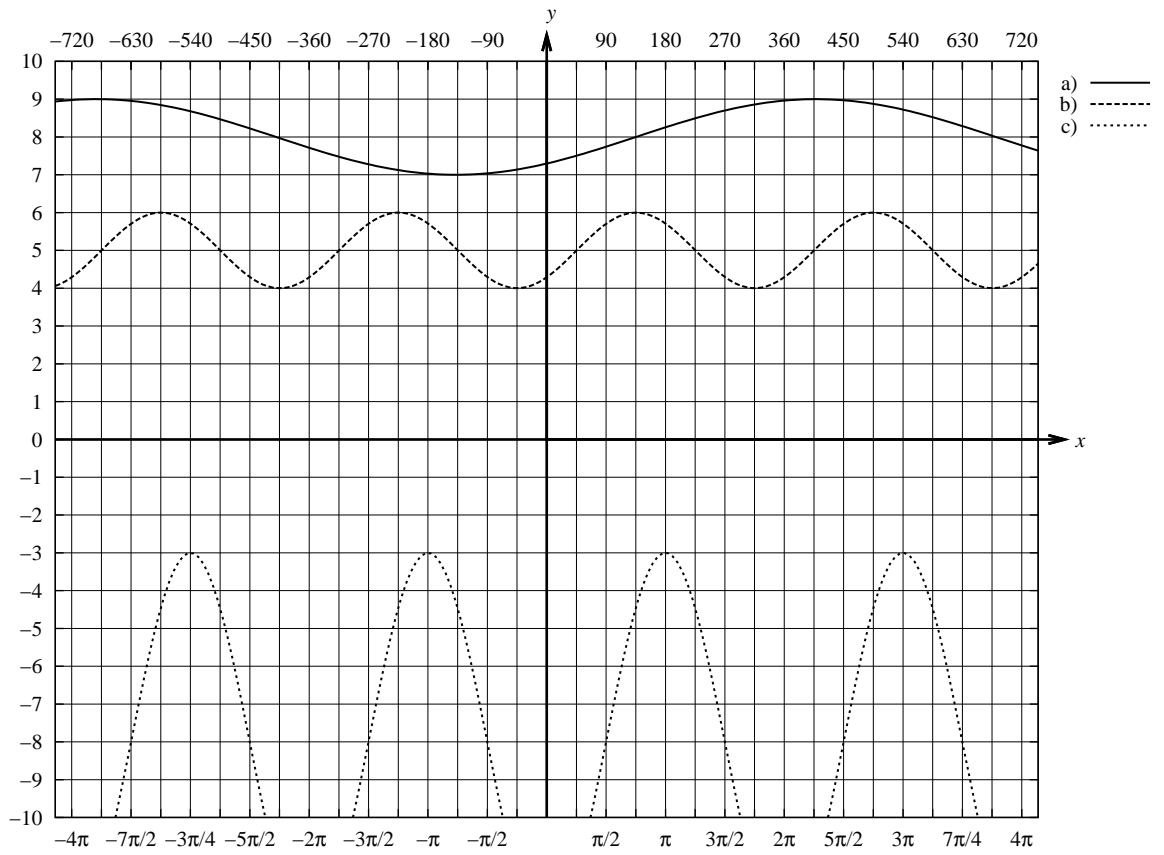


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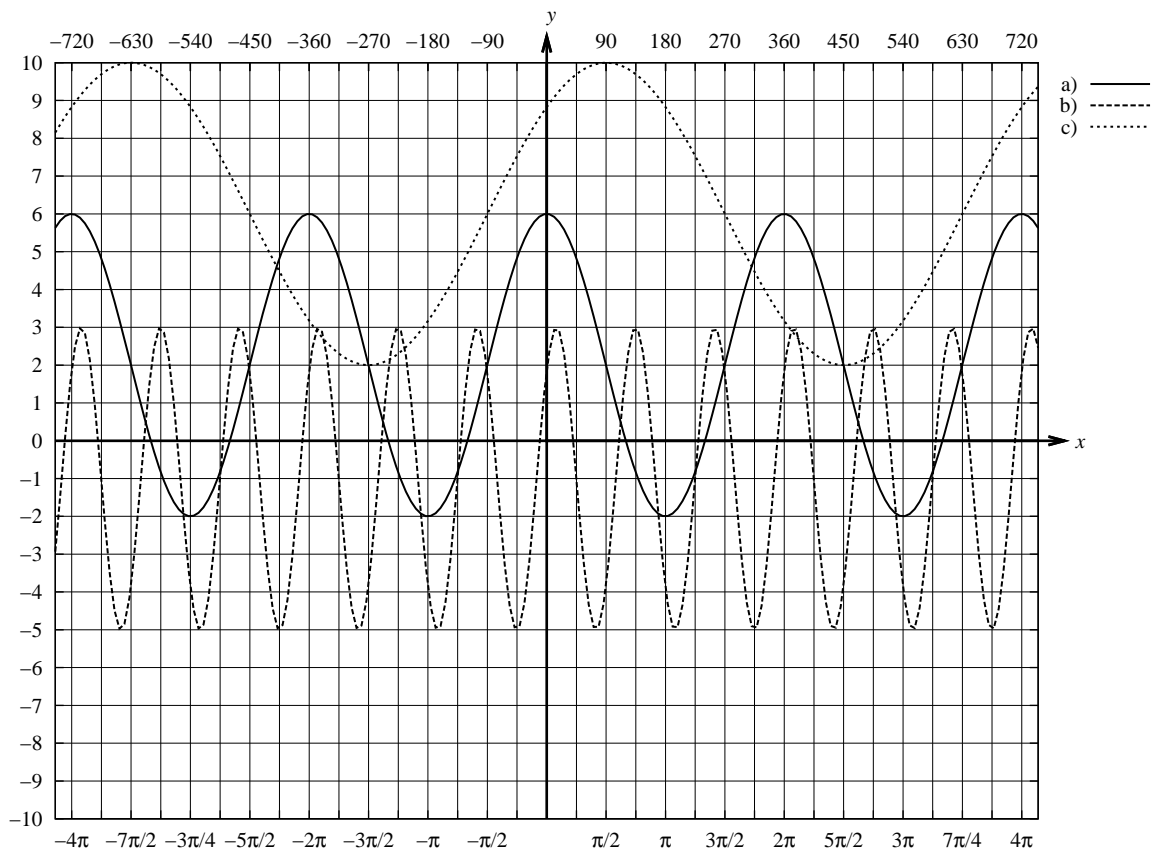


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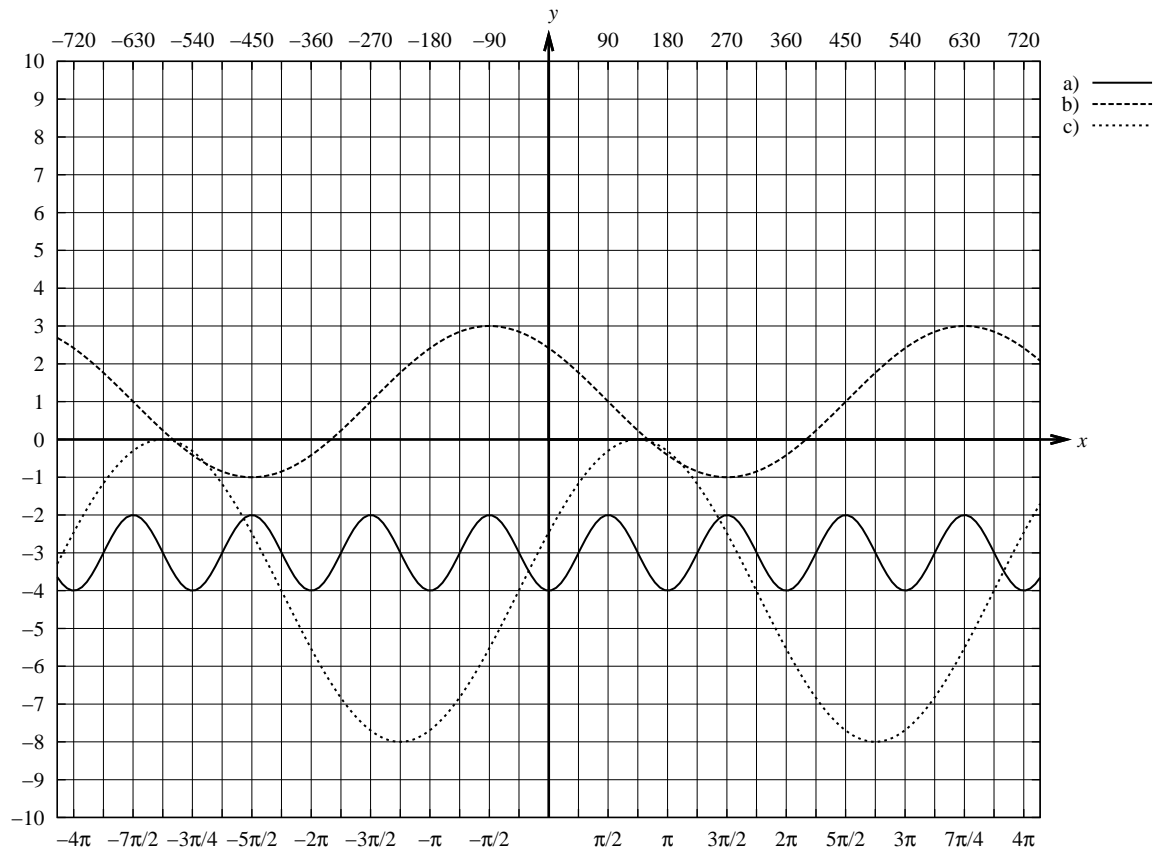


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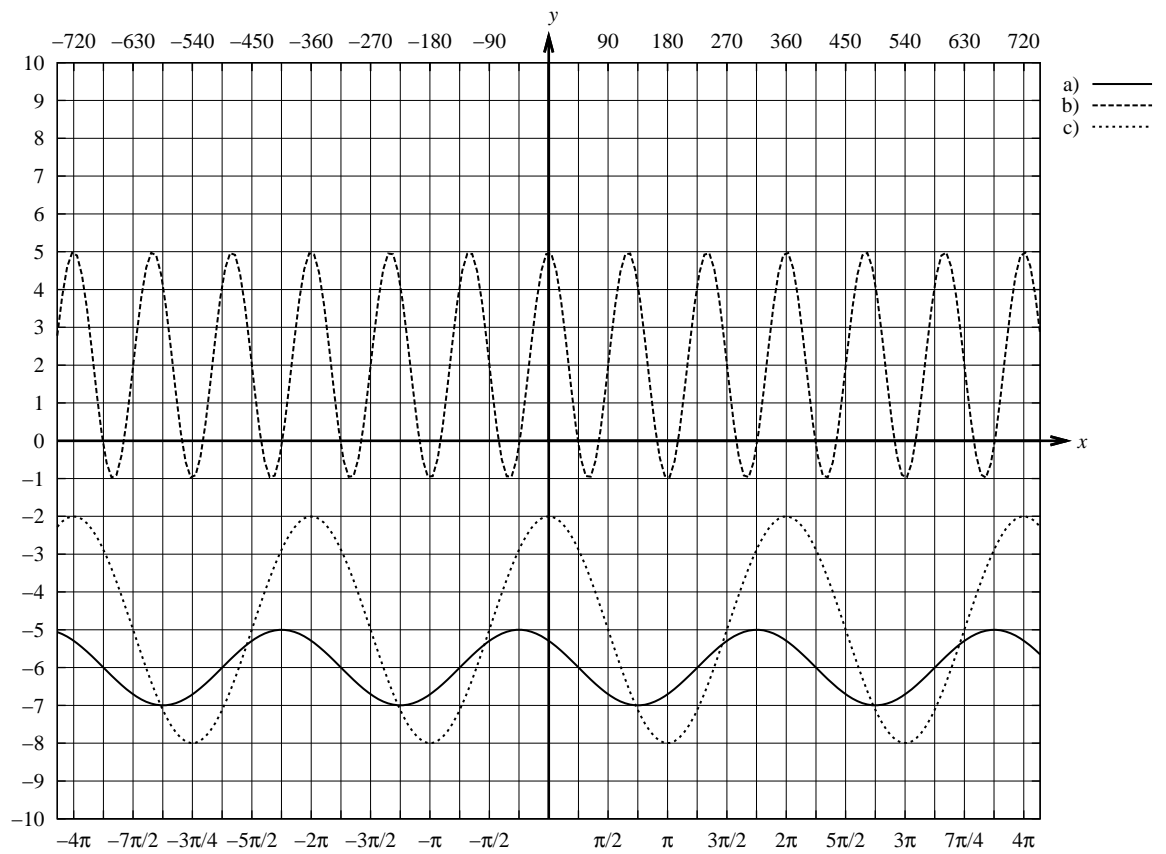


9. a) b) c)

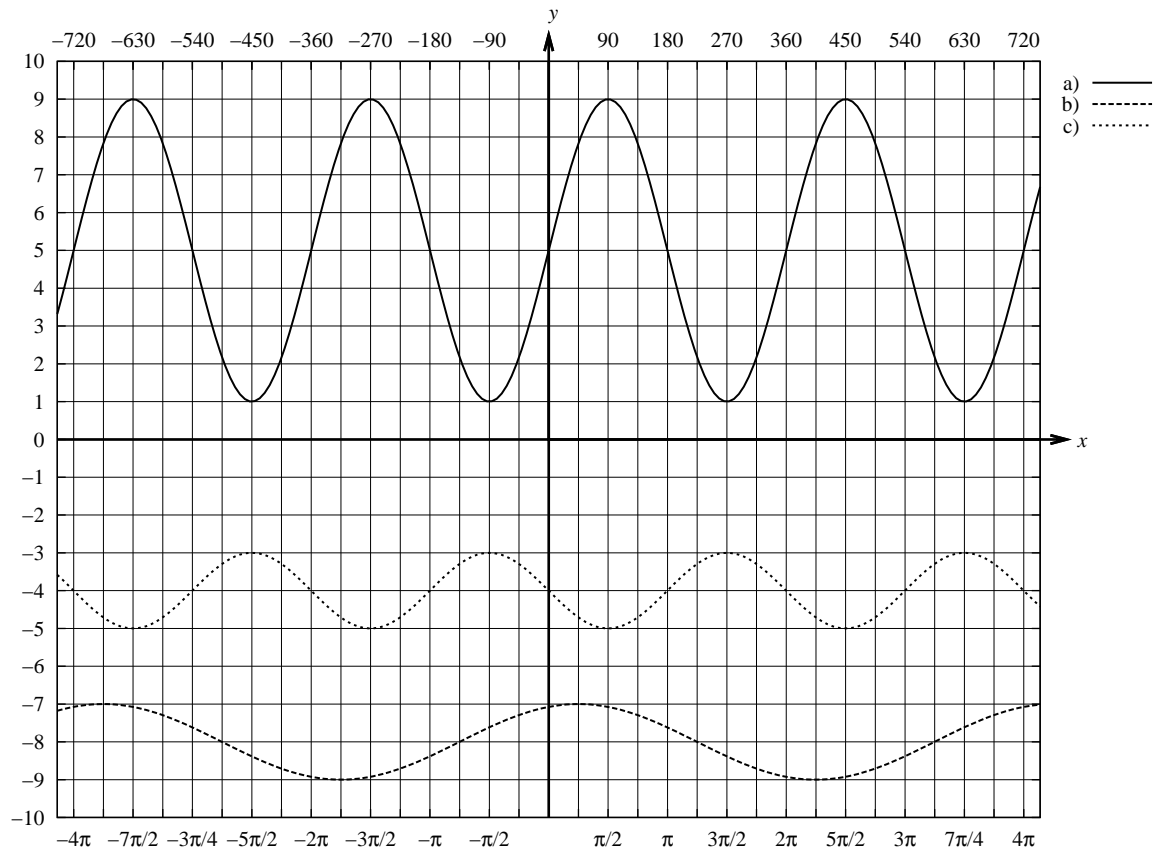
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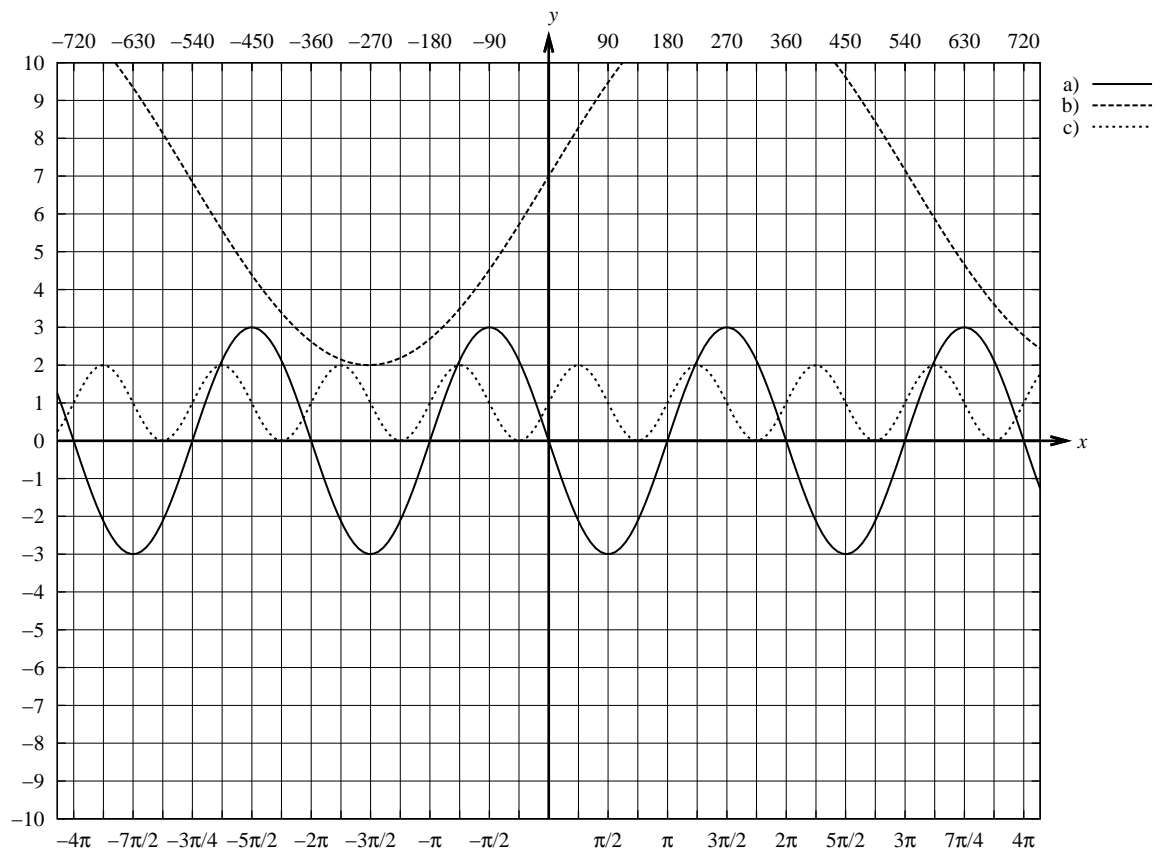
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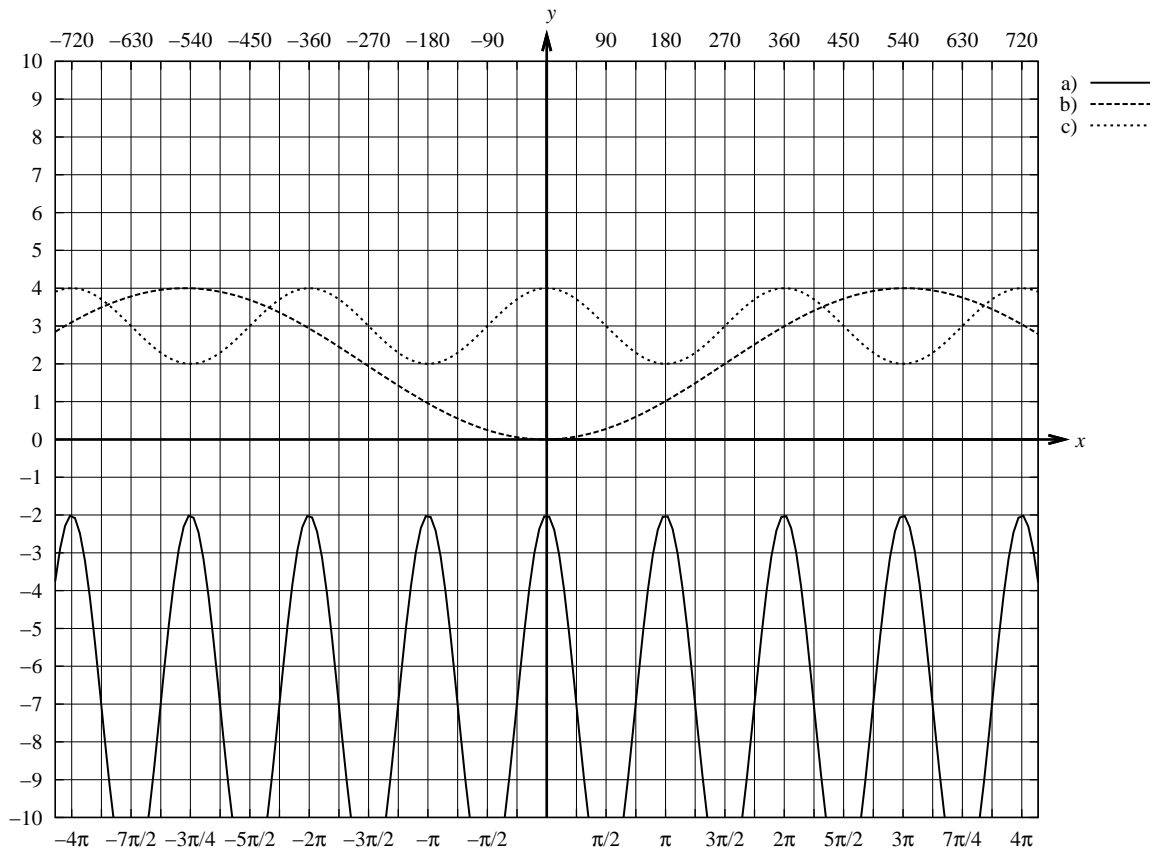
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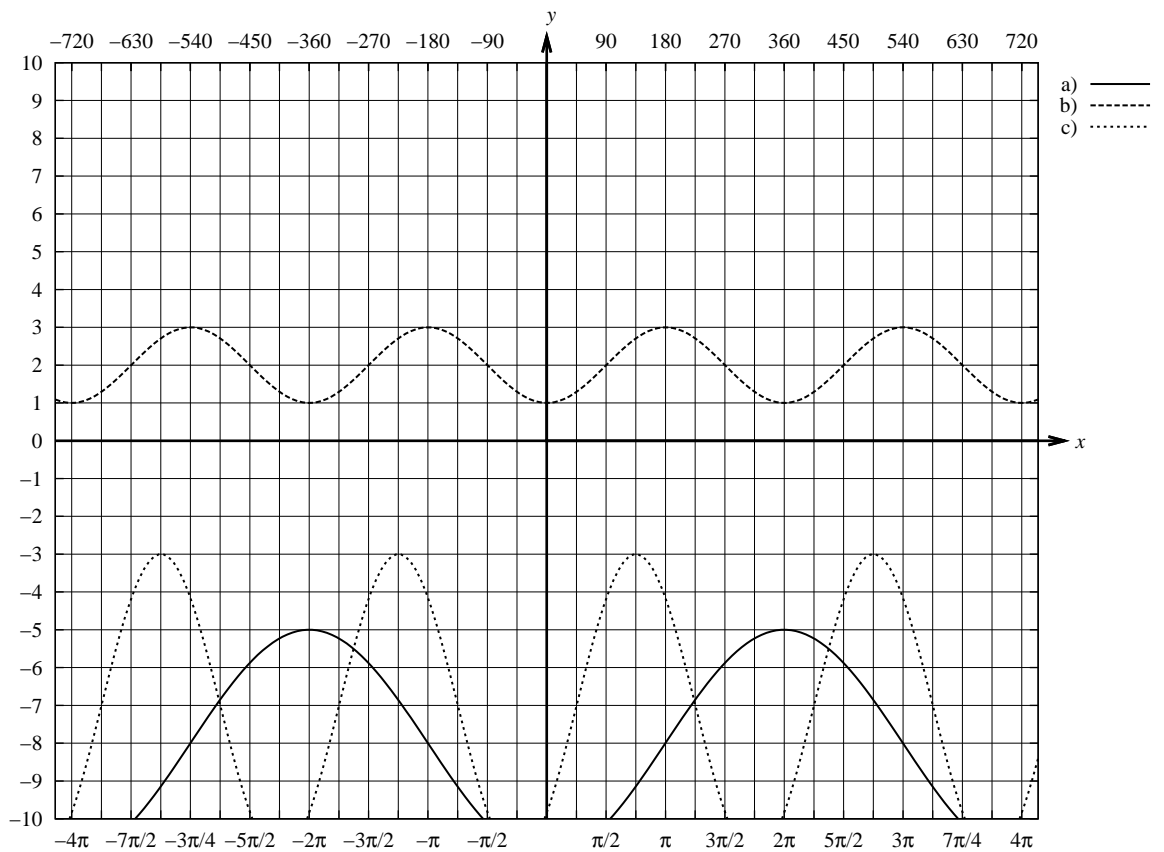
13. a) b) c)



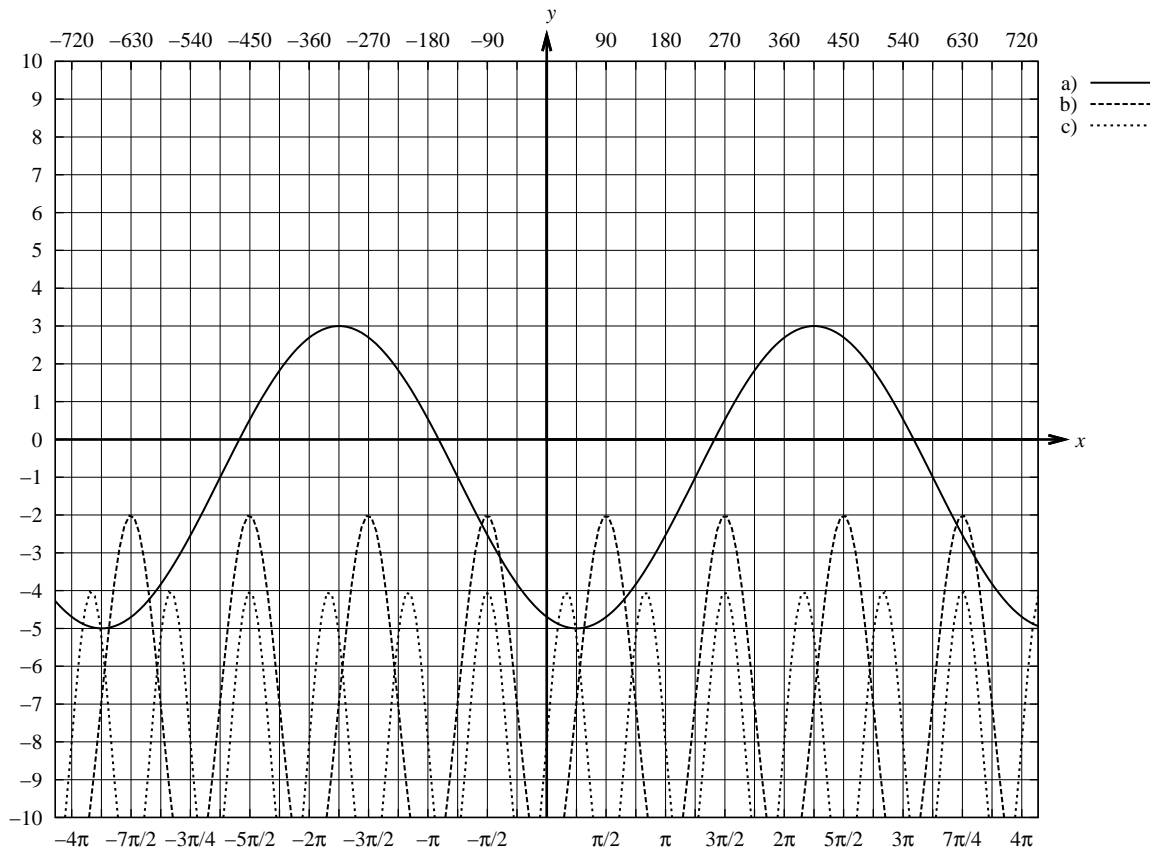
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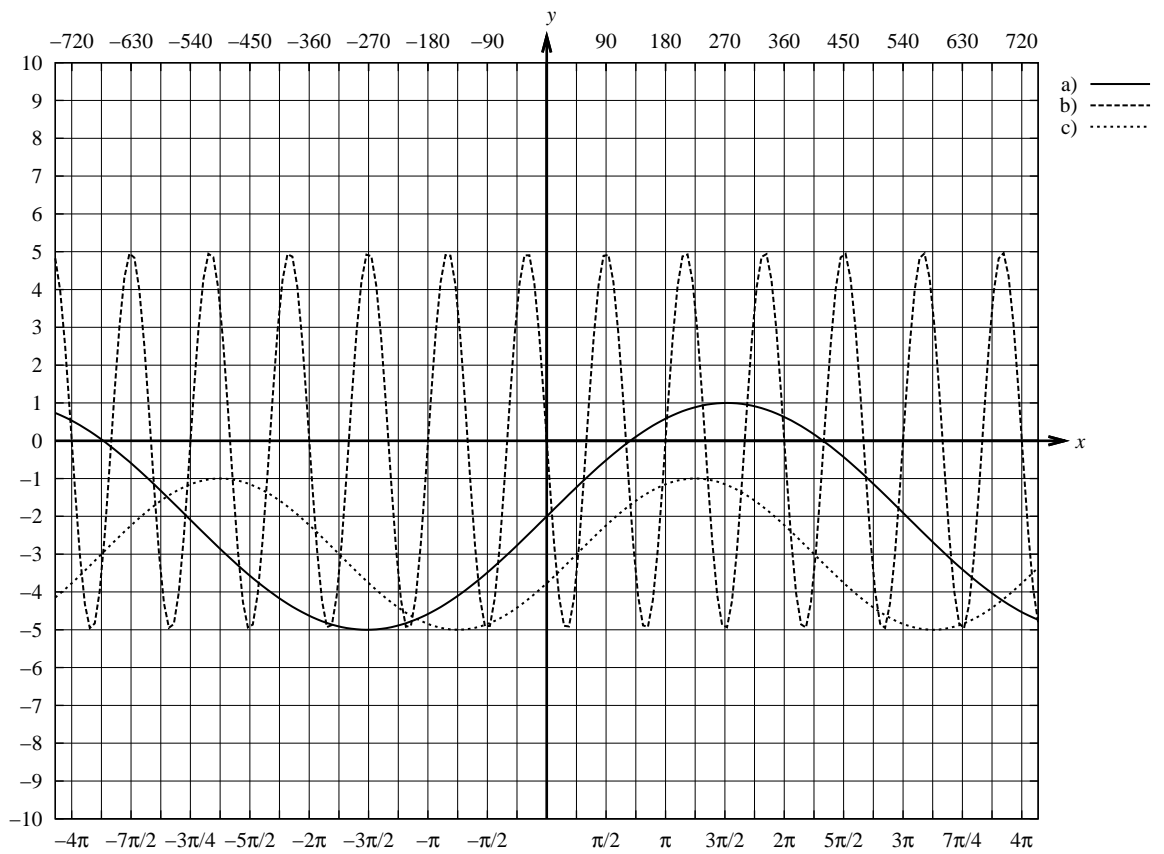
15. a) b) c)



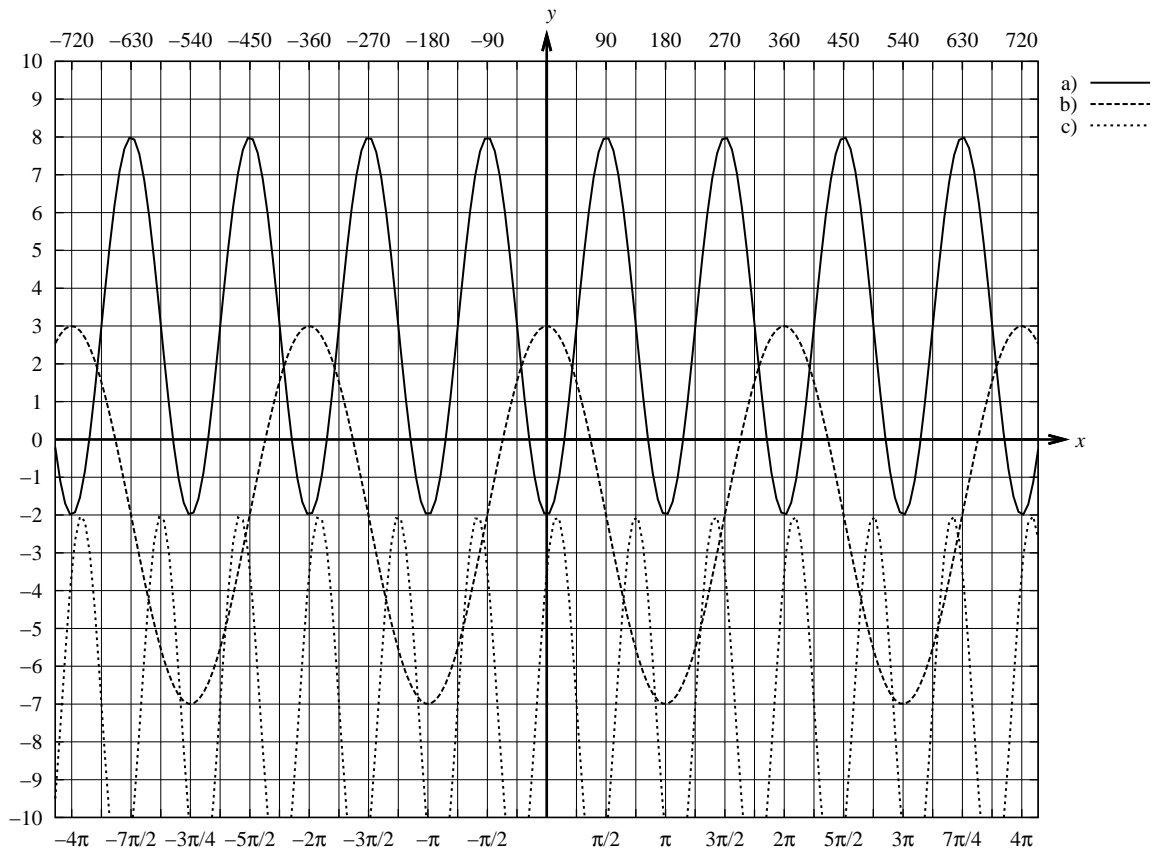
16. a) b) c)



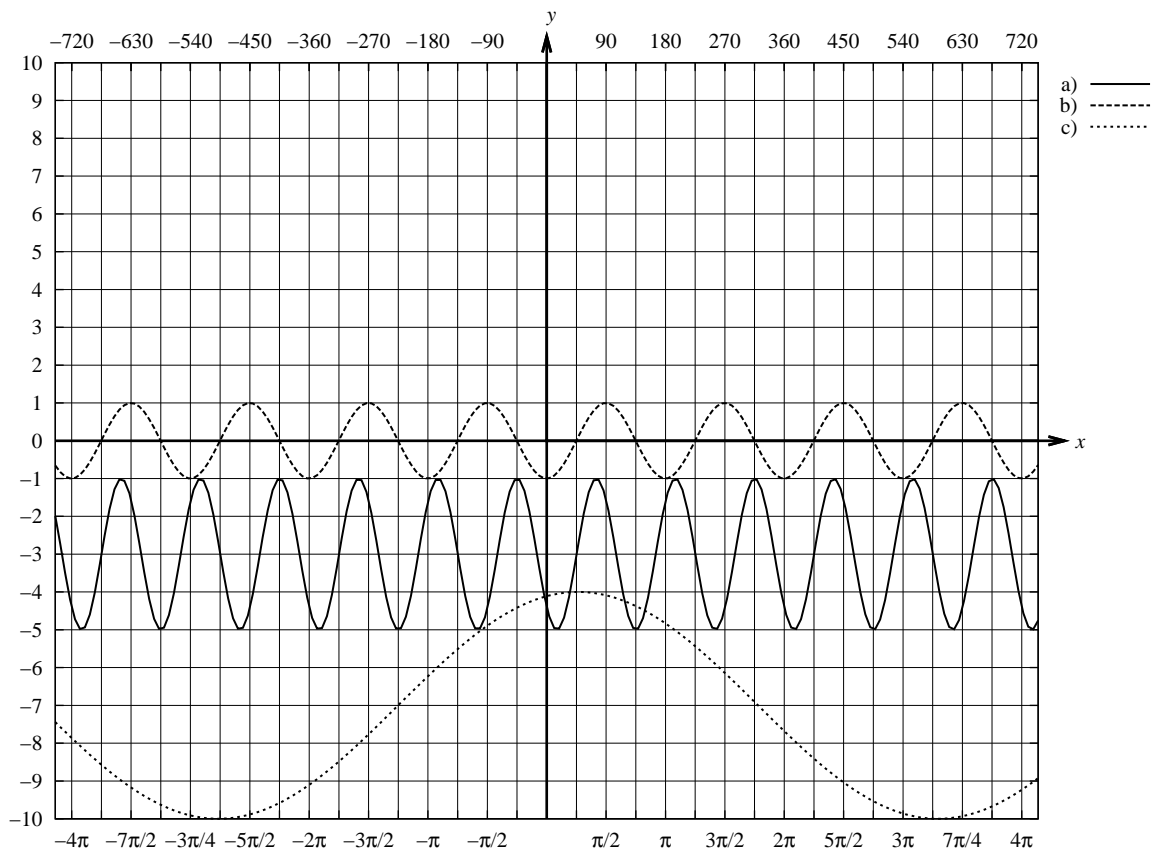
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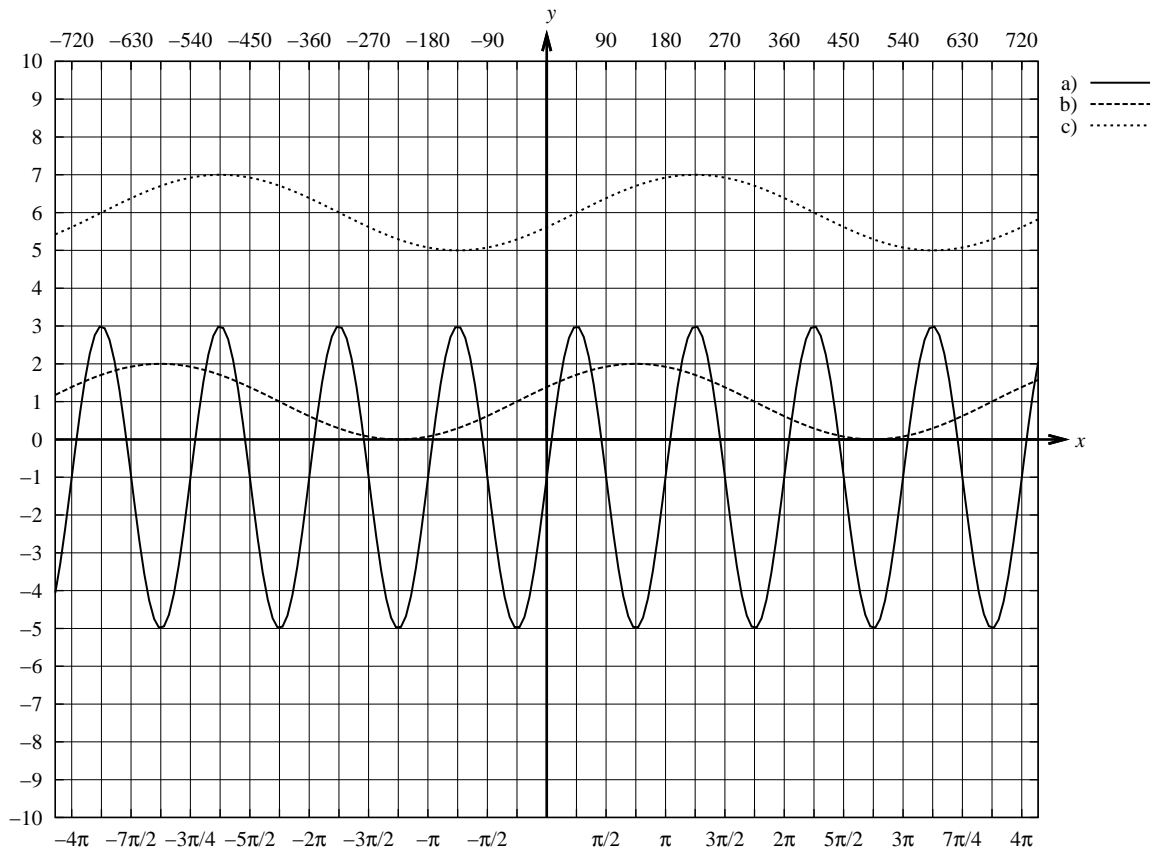
18. a) b) c)



19. a) b) c)



20. a) b) c)



21. a) b) c)

