



ÖVNINGAR PÅ ODE AV GRAD 1

EUROPASKOLAN
STRÄNGNÄS

JOHAN WILD

2010-02-14

©Johan Wild johan.wild@europaskolan.se Får gärna användas i undervisning, kontakta i så fall författaren.

Lös följande ODE.

1. $y' - \frac{7}{8}y = \frac{5}{3}\sin(6x)$ $y(5) = -8$
2. $y' - \frac{9}{8}y = -\frac{9}{5}x + \frac{4}{3}$ $y(8) = 8$
3. $y' - 9y = \frac{1}{9}x^2 - x - 1$ $y(-5) = -1$
4. $y' - \frac{5}{3}y = 1$ $y(3) = -6$
5. $y' + \frac{9}{8}y = -\frac{7}{3}\sin(9x) + \frac{4}{7}\cos(9x)$ $y(-1) = 6$
6. $y' + y = \frac{1}{4}x^2 + \frac{7}{5}x + 2$ $y(-5) = 6$
7. $y' - y = -\frac{1}{5}$ $y(5) = 0$
8. $y' - \frac{2}{5}y = \frac{4}{9}x + \frac{1}{2}$ $y(-6) = 0$
9. $y' + \frac{9}{7}y = \frac{1}{8}\sin(9x) - \cos(9x)$ $y(1) = -1$
10. $y' - \frac{4}{5}y = 1$ $y(6) = 0$
11. $y' + \frac{1}{4}y = \frac{1}{2}x$ $y(-3) = -3$
12. $y' + \frac{1}{6}y = \frac{2}{5}x^2 - 3x - 1$ $y(7) = 6$
13. $y' + \frac{3}{4}y = -\frac{5}{8}\sin(x) + 3\cos(x)$ $y(-9) = 5$
14. $y' - 5y = -4x + \frac{1}{2}$ $y(-4) = 4$
15. $y' - \frac{1}{3}y = -\frac{1}{2}x - 6$ $y(8) = 0$
16. $y' - y = 2$ $y(4) = 3$
17. $y' + \frac{1}{2}y = \frac{9}{8}\sin(6x)$ $y(-2) = 1$
18. $y' - \frac{1}{2}y = -\frac{8}{3}x^2 + \frac{9}{8}x - \frac{1}{2}$ $y(-4) = 8$
19. $y' + \frac{5}{4}y = \frac{2}{3}$ $y(1) = -6$
20. $y' + \frac{7}{9}y = -\frac{1}{2}x - \frac{6}{7}$ $y(-1) = -6$
21. $y' - \frac{4}{3}y = -\frac{4}{5}\sin(9x) - \frac{3}{8}\cos(9x)$ $y(-9) = -6$

Facit:

1. $y_h(x) = \left(-8 + \frac{280}{7059} \sin(6 \cdot 5) + \frac{640}{2353} \cos(6 \cdot 5)\right) e^{-\frac{35}{8}} \cdot e^{\frac{7}{8}x}$
 $y_p(x) = -\frac{280}{7059} \sin(6x) - \frac{640}{2353} \cos(6x)$
 $y(x) \approx -0.10067034e^{\frac{7}{8}x} - 0.03966568 \sin(6x) - 0.27199320 \cos(6x)$
2. $y_h(x) = -\frac{136}{27}e^{-9} \cdot e^{\frac{9}{8}x}$ $y_p(x) = \frac{8}{5}x + \frac{32}{135}$
3. $y_h(x) = -\frac{1789}{6561}e^{45} \cdot e^{9x}$ $y_p(x) = -\frac{1}{81}x^2 + \frac{79}{729}x + \frac{808}{6561}$
4. $y_h(x) = -\frac{27}{5}e^{-5} \cdot e^{\frac{5}{3}x}$ $y_p(x) = -\frac{3}{5}$
5. $y_h(x) = \left(6 - \frac{376}{12285} \sin(9 \cdot -1) - \frac{3232}{12285} \cos(9 \cdot -1)\right) e^{-\frac{9}{8}} \cdot e^{-\frac{9}{8}x}$
 $y_p(x) = \frac{376}{12285} \sin(9x) + \frac{3232}{12285} \cos(9x)$
 $y(x) \approx 2.02983054e^{-\frac{9}{8}x} + 0.03060643 \sin(9x) + 0.26308506 \cos(9x)$
6. $y_h(x) = \frac{63}{20}e^{-5} \cdot e^{-x}$ $y_p(x) = \frac{1}{4}x^2 + \frac{9}{10}x + \frac{11}{10}$
7. $y_h(x) = -\frac{1}{5}e^{-5} \cdot e^x$ $y_p(x) = \frac{1}{5}$
8. $y_h(x) = -\frac{95}{36}e^{\frac{12}{5}} \cdot e^{\frac{2}{5}x}$ $y_p(x) = -\frac{10}{9}x - \frac{145}{36}$
9. $y_h(x) = \left(-1 + \frac{77}{720} \sin(9 \cdot 1) + \frac{7}{240} \cos(9 \cdot 1)\right) e^{\frac{9}{7}} \cdot e^{-\frac{9}{7}x}$
 $y_p(x) = -\frac{77}{720} \sin(9x) - \frac{7}{240} \cos(9x)$
 $y(x) \approx -3.55395197e^{-\frac{9}{7}x} - 0.10694444 \sin(9x) - 0.02916667 \cos(9x)$
10. $y_h(x) = \frac{5}{4}e^{-\frac{24}{5}} \cdot e^{\frac{4}{5}x}$ $y_p(x) = -\frac{5}{4}$
11. $y_h(x) = 11e^{-\frac{3}{4}} \cdot e^{-\frac{1}{4}x}$ $y_p(x) = 2x - 8$
12. $y_h(x) = -\frac{294}{5}e^{\frac{7}{6}} \cdot e^{-\frac{1}{6}x}$ $y_p(x) = \frac{12}{5}x^2 - \frac{234}{5}x + \frac{1374}{5}$
13. $y_h(x) = \left(5 - \frac{81}{50} \sin(\cdot - 9) - \frac{46}{25} \cos(\cdot - 9)\right) e^{-\frac{27}{4}} \cdot e^{-\frac{3}{4}x}$
 $y_p(x) = \frac{81}{50} \sin(x) + \frac{46}{25} \cos(x)$
 $y(x) \approx 0.00859907e^{-\frac{3}{4}x} + 1.62000000 \sin(1x) + 1.84000000 \cos(1x)$
14. $y_h(x) = \frac{357}{50}e^{20} \cdot e^{5x}$ $y_p(x) = \frac{4}{5}x + \frac{3}{50}$
15. $y_h(x) = -\frac{69}{2}e^{-\frac{8}{3}} \cdot e^{\frac{1}{3}x}$ $y_p(x) = \frac{3}{2}x + \frac{45}{2}$
16. $y_h(x) = 5e^{-4} \cdot e^x$ $y_p(x) = -2$
17. $y_h(x) = \left(1 - \frac{9}{580} \sin(6 \cdot -2) + \frac{27}{145} \cos(6 \cdot -2)\right) e^{-1} \cdot e^{-\frac{1}{2}x}$
 $y_p(x) = \frac{9}{580} \sin(6x) - \frac{27}{145} \cos(6x)$
 $y(x) \approx 0.42262185e^{-\frac{1}{2}x} + 0.01551724 \sin(6x) - 0.18620690 \cos(6x)$
18. $y_h(x) = -\frac{241}{6}e^2 \cdot e^{\frac{1}{2}x}$ $y_p(x) = \frac{16}{3}x^2 + \frac{229}{12}x + \frac{235}{6}$
19. $y_h(x) = -\frac{98}{15}e^{\frac{5}{4}} \cdot e^{-\frac{5}{4}x}$ $y_p(x) = \frac{8}{15}$
20. $y_h(x) = -\frac{312}{49}e^{-\frac{7}{9}} \cdot e^{-\frac{7}{9}x}$ $y_p(x) = -\frac{9}{14}x - \frac{27}{98}$
21. $y_h(x) = \left(-6 + \frac{831}{29800} \sin(9 \cdot -9) - \frac{693}{7450} \cos(9 \cdot -9)\right) e^{12} \cdot e^{\frac{4}{3}x}$
 $y_p(x) = -\frac{831}{29800} \sin(9x) + \frac{693}{7450} \cos(9x)$
 $y(x) \approx -985428.57712215e^{\frac{4}{3}x} - 0.02788591 \sin(9x) + 0.09302013 \cos(9x)$